Fractal Analysis of Complex Power Load Variations Through Adaptive Multiscale Filtering

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***Abstract*—Power load analysis is important for optimizing resource allocation, planning the production of electricity, and predicting power markets. Yet, it is challenging, since load data exhibit both periodic and stochastic features, and is affected by a multitude of factors including social, economic, political, and climatic factors, as well as industrial structure, living standards, and user behaviors. In this paper, we employ a multiscale framework to systematically analyze load data from two electric utilities in two cities of different size in China. The low frequency trend signals in both load data sets are quite irregular. The detrended data of the load time series are further denoised to remove high frequency noise. Fourier spectral analysis of the original and filtered data shows that the load time series has very strong spectral peaks corresponding to a period of one day. Using adaptive fractal analysis, which can best extract fractal behaviors from signals with strong oscillatory trends, we further show that load time series has long-range correlations. Amazingly, maxima of the temporal variations of the long-range correlations correspond well with temperature minima, highlighting that long-range correlations are stronger in winter than in summer.**

Keywords—power load variation; fractals; multiscale analysis

I. INTRODUCTION

Comprising interwoven subsystems such as electric utilities, power grids, and their users, electrical systems are of fundamental importance to modern society. They are dissipative nonlinear dynamical systems with characteristics such as openness and nonuniformity, and may exhibit complex behaviors including bifurcations, catastrophe, and possibly chaos. Among the many fundamental issues associated with electrical systems are the characterization, modeling, and prediction of power load time series. Solutions to these issues are the basis for optimization of resource allocation, planning of the production of electricity, and prediction of the power markets. Analysis of power load time series is very challenging, however, since load time series is highly complicated, exhibiting both periodic and stochastic features. Such behaviors are generic of multiscale systems [1]. The complexity of load time series is further enhanced by social, economic, political, and climatic factors. Coupled with grid structure, industrial structure, living standards, and user behaviors, load time series varies from one region to another significantly.

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In recent years, there have been considerable efforts to model power load time series using chaos [2] and fractal theories [2–8]. There also have been efforts to study the power markets using multifractal theory [9, 10] and by explicitly considering temporal scales [11]. However, some of the researches published are not very satisfactory. For example, the problem is related to fractal analysis of load time series. A typical high frequency load time series contains periodic components, such as the daily (or diurnal) cycle. Fractal, by definition, is without any scale. How can periodicity and fractal coexist in load time series, and what effect will periodicity have on the estimation of fractal scaling exponents? The another question is also related to fractal characterization of load time series. In many fractal-based forecasting of load time series (see e.g., [3, 6, 8]), the value of the fractal scaling parameter (called the Hurst parameter) is given instead of estimated from measured data. This amounts to modeling load time series by purely fractal processes without incorporating any other features that are characteristic of load time series. The fundamental question is whether this is reasonable.

To gain insights into the above fundamental questions, in this paper, we employ an adaptive multiscale filtering based approach to systematically analyze load time series from two electric utilities in two cities of different size in China. In particular, we will examine whether features such as periodicity, chaos, and fractals may coexist in load time series, and if yes, how one feature may affect characterization of other features. We will also examine factors that may cause temporal variations of those features.

II. JOINT CHAOS AND FRACTAL ANALYSIS OF LOAD VARIATION USING MULTISCALE APPROACH

A. Load time series data

The load time series data analyzed here were obtained from two electric utilities in two different cities, Guilin and Leshan, of Guangxi Province, which is in southwest China. Guilin is a middle sized city, most well-known for tourism. Leshan is a smaller city. The data from Guilin covers a time span of more than 4 years, from January 1, 2005, to April 29, 2010. The data from Leshan covers a time span of two years, from January 1, 2010, to December 31, 2011. The sampling time is 15 minutes for both data sets, and hence, there are 96 points on each day.

The raw load time series data are shown as the blue curves in Figs. 1 and 2. We observe that both are very irregular.

B. Adaptive multiscale decomposition of the load variation

To better understand what have contributed to the complexity of load time series, we employ a recently developed adaptive method, which is better for removing noise and determining trends than existing methods such as chaos-based or wavelet-based approaches [12–14].

The method works as follows. It first partitions a time series into segments (or windows) of length $w = 2n+1$ points, where neighboring segments overlap by $n+1$ points. While this has ensured symmetry, it also introduces a time scale of $(w+1)\tau/2 = (n+1)\tau$, where τ is the sampling time. For each segment, we fit a best polynomial of order M. Note that $M = 0$ and 1 correspond to piece-wise constant and linear fitting, respectively. Denote the fitted polynomial for the i-th and $(i+1)$-th segments by $y^{(i)}(l_1), y^{(i+1)}(l_2), l_1, l_2 = 1, \cdots, 2n+1$, respectively. Note the length of the last segment may be smaller than $2n+1$. We define the fitting for the overlapped region as

$$y^{(c)}(l) = w_1 y^{(i)}(l+n) + w_2 y^{(i+1)}(l), = 1, 2, \cdots, n+1 \tag{1}$$

where $w_1 = (1\text{-}\frac{l-1}{n})$, $w_2 = \frac{l-1}{n}$ can be written as $(1 - d_j / \mathrm{n})$, $j = 1, 2$, where d_j denotes the distances between the point and the centers of $y^{(i)}$ and $y^{(i+1)}$, respectively. This means the weights decrease linearly with the distance between the point and the center of the segment. Such a weighting ensures symmetry and effectively eliminates any jumps or discontinuities around the boundaries of neighboring segments. In fact, the scheme ensures that the fitting is continuous everywhere, is smooth at the non-boundary points, and has the right- and left-derivatives at the boundary. The method can effectively determine any kind of trend signal. Those for the load time series depicted as blue curves in Figs. 1(a) and 2(a) are shown there as red curves. They are determined with a window size of 699 sample points and a polynomial order of 2.

For convenience of further analysis, we denote the raw load time series by $x(t)$, and the trend signals by *trend*(t). Then the detrended signals are

$$x_{detrended} = x(t) - trend(t) \tag{2}$$

To better see how $x_{detrended}$ looks like, we have shown in Figs. 1(b) and 2(b) as blue curves a small segment of the detrended data. Both are oscillatory. However, their waveforms are not the same, suggesting that the signals from different electric utilities are very different. We also observe that the detrended signals are noisy. This high frequency noise can also be neatly removed by the adaptive filter employed here. This is achieved by using the filter with a small window size and determine another trend. With a window size of 9 sample points and a polynomial order of 2, we obtain the new trend signals shown as red in Figs. 1(b) and 2(b). This signal may be called detrended and denoised signal, i.e., band-passed signal. Note that the oscillatory feature of the load time series is better revealed by this band-pass signal. The difference between the blue and the red curves in Figs. 1(b) and 2(b) is the high frequency noise. This noise is smaller for Fig. 2(b), suggesting that high frequency noise in load time series from smaller electric utilities is weaker. This is consistent with the fact that load time series from smaller electric utilities has less uncertainty.

We have computed the power-spectral density (PSD) for the raw, detrended, as well as band-pass signals. They are shown as blue, red, and green curves in Figs. 1(d) and 2(d), respectively. The most salient features of both figures are the sharp spectral peaks corresponding to 1 day and its harmonics. Note that most of the blue curves are covered by the red and the green colors, except at the frequency close to 0 Hz. They are contributed by the red trend signals shown in Figs. 1(a) and 2(a). We also note that the difference between the red and the green curves in Fig. 1(d) is quite big, but negligible in Fig. 2(d). This is consistent with our earlier observation that high frequency noise is stronger in load time series from the Guilin.

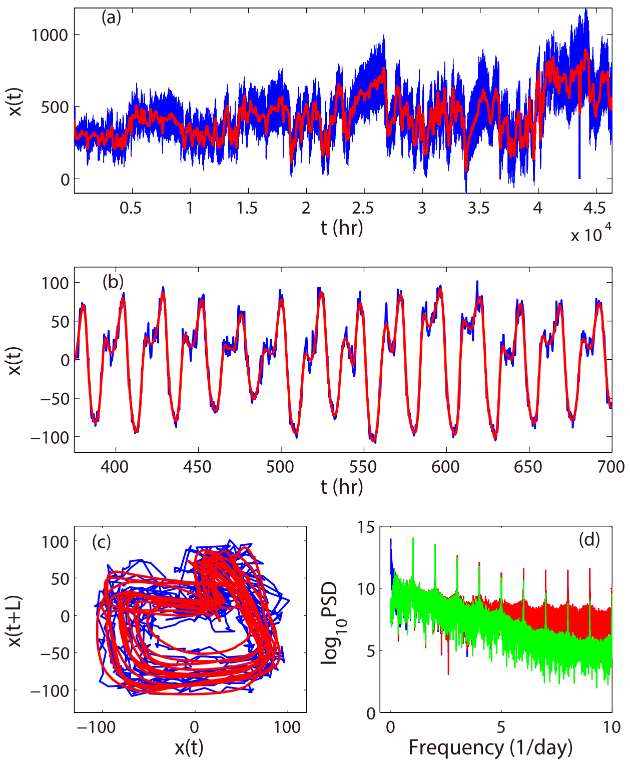

Figure 1: (a) Raw load series data (blue) and the low frequency trend signal (red) from the city of Guilin; (b) a short segment of the detrended data (blue) and the denoised data (red); (c) 2-D phase diagrams corresponding to the data shown in (b); (d) Power spectral density (PSD) curves for the raw data (blue), detrended data (red), and also denoised data (green)

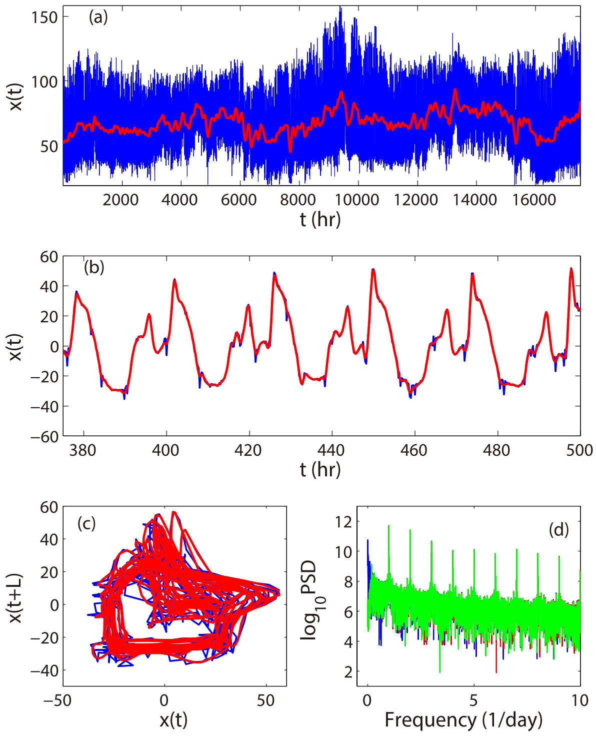

Figure 2: Similar to Fig. 1 except for the city of Leshan.

C. *Adaptive fractal analysis of the power load data*

we proceed to better characterize the random amplitude variations by random fractal theory [1]. One of the main models in random fractal theory is the $1/f^{\alpha}$ processes, where $\alpha = 2$ corresponds to the standard Brownian motion. Activities of many complex systems are characterized by such processes. A sub-class of such processes, denoted as $1/f^{2H+1}$, is called processes with long-range correlations (or long memories) characterized by a Hurst parameter H. Depending on whether $0 < H < 1/2$, $H = 1/2$, or $1/2 < H < 1$ [15], they are said to have antipersistent correlations, memoryless or only short-range correlations, or persistent long-range correlations. Prominent examples of such processes include vision [16], finance [17], DNA sequences [18–22], human cognition [23] and coordination [24], posture [25], cardiac dynamics [26–29], as well as the distribution of prime numbers [30], to name but a few.

Before we proceed to estimate H (or equivalently α, since $\alpha = 2H+1$ from the power load data, we provide some mathematical details so that we can better understand the meaning of H. Let $\{x_1, x_2, \cdots, x_n\}$ be a stationary stochastic process with mean x and autocorrelation function of the type,

$$r(k) \sim k^{2H-2}, \ as\ k \to \infty \tag{3}$$

where 0 < H < 1 is the Hurst parameter. When $1/2 < H < 1$, $\sum_k r(k) = \infty$, leading to the term long range correlation. $\{x_1, x_2, \cdots, x_n\}$ is often called an increment (or noise) process. Its power spectral density (PSD) is $1/f^{2H+1}$. Its integration,

$$u(i) = \sum_{k=1}^{i} (x_k - \bar{x}),\ i = 1, 2, \cdots, n, \tag{4}$$

is called a random walk process having PSD $1/f^{2H+1}$. Being $1/f$ processes, they cannot be aptly modeled by Markov processes or ARIMA models [31], since the PSD for those processes are distinctly different from $1/f$. To adequately model $1/f$ processes, fractional order processes have to be used. The most popular is the fractional Brownian motion model [15].

To deepen our understanding of the Hurst parameter, let us smooth $\{x_1, x_2, \cdots, x_n\}$ using non-overlapping windows to yield a new time series,

$$X_t^{(m)} = (x_{tm-m+1} + \cdots + x_{tm})/m,\ t \ge 1 \tag{5}$$

It can be proven that the variance of the new time series is given by [1, 32]

$$var(X_t^{(m)}) = \sigma^2 m^{2H-2} \tag{6}$$

where σ^2 is the variance of original stochastic process $\{x_1, x_2, \cdots, x_n\}$. Eq. (6) offers an excellent means of understanding H. For example, if $H = 0.50$, $m = 100$, then $var(X^{(m)}) = \sigma^2/100$. When $H = 0.75$, in order to have $var(X^{(m)}) = \sigma^2/100$, then we need $m = 10^4$, which is much larger than m = 100 for the case of $H = 0.50$. On the other hand, when $H = 0.50$, if we still want $var(X^{(m)}) = \sigma^2/100$, then $m \approx 21.5$, much smaller than $m = 100$, the case of $H = 0.50$. An interesting lesson from such a simple discussion is that if a time series is short while its H is close to 1, then smoothing is not a viable option for reducing the variations there.

Let us now proceed to estimate H. It is a highly nontrivial task, however, since our power load data contain a very strong oscillatory component — fractal signals do not possess any temporal or spatial scales, while oscillatory motions have a well-defined time scale, therefore, they are entirely different types of signals. More seriously, the oscillatory components in the load data prevent well established methods for estimating the Hurst parameter from being effective here. Fortunately, the adaptive algorithm already used earlier offers a way out. The method is called adaptive fractal analysis (AFA) [33–36]. It works as follows. For a window size w, we determine, for the random walk-like noise process $n(i)$ a global trend $v(i), i = 1, 2, \cdots N$. Here N is the length of the random walk process. The residual, $n(i) - v(i)$, characterizes fluctuations around the global trend, and its variance yields the Hurst parameter H [33],

$$F^{(2)}(w) = [\frac{1}{N}\sum_{i=1}^{N}(n(i) - v(i))^2 \sim w^H \quad (7)$$

Recall that the power load data are quite nonstationary. To examine whether the main fractal features may vary with time or not, it is important to first partition the load data into short segments, then estimate the Hurst parameter for each segment. We have chosen the segment length to be $96 \times 30 = 2880$ points, i.e., one month, since there are 96 sample points on each day. To improve the resolution, we have made the adjacent segments to overlap by half of the window length. An example of AFA, for an arbitrarily chosen window is shown in Fig. 3(a). The curve is linear up to $w = 2^7$ sample points, which is slightly longer than a day. The slope for the linear part of the curve yields the Hurst parameter *H*. Its variation with time is shown in Fig. 3(b) as the red curve. We observe that it has an oscillatory feature, with a period of about 1 year. This suggests that this variation may be correlated with the yearly temperature variation. To check this idea, we have also plotted in Fig. 3(b) a curve obtained by rescaling the local temperature at the city of Guilin by a factor of 0.01, then shifting upward by 0.5, i.e.,

$$T' = T / 100 + 0.5 \quad (8)$$

The transformed temperature is plotted in the figure as the black curve. Amazingly, we observe that maxima of the $H(t)$ curve correspond to minima of the curve for the transformed temperature. In other words, the power load data have stronger long range correlations during winter times.

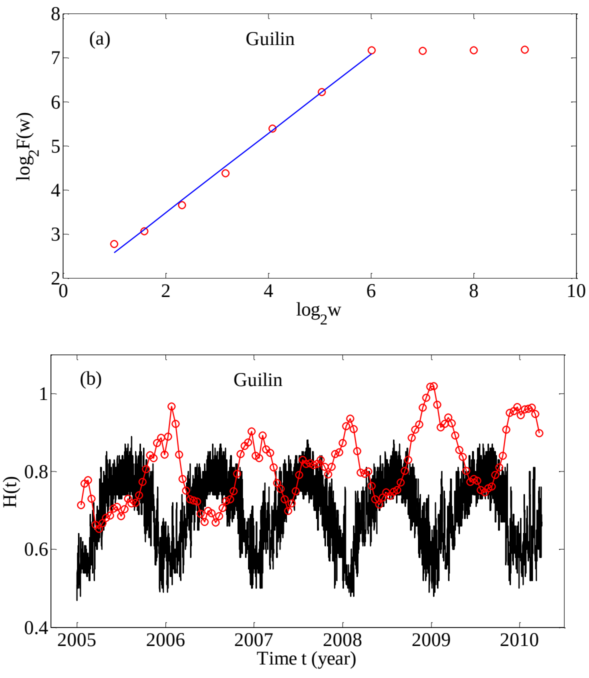

Figure 3: (a) An example of log2 F(w) vs. log2 w for the load series of Guilin, illustrating AFA, and (b) the variation of H(t) vs. t (red). To correlate with temperature variation, a curve (in black) is also plotted based on the local temperature data (see text for details).

Similar analysis has been carried out to the load data for the city of Leshan. The results are shown in Fig. 4. We observe very similar results; in particular, $H(t)$ are larger during winter times, and therefore, again the load data have stronger long range correlations in winter.

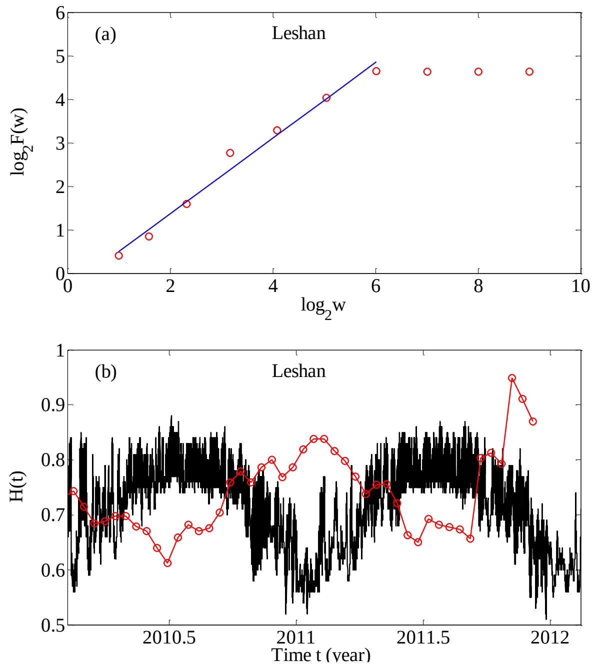

Figure 4: (a) An example of log2 F(w) vs. log2 w for the load series of Leshan, illustrating AFA, and (b) the variation of H(t) vs. t (red). To correlate with temperature variation, a curve (in black) is also plotted based on the local temperature data (see text for details).

III. Conclusion and Discussion

Analysis of power load variation is an important problem in Engineering, since it is the basis for the optimization of resource allocation, planning of the production of electricity, and prediction of power markets. Yet, it is challenging, as load variation is affected by many factors including social, economic, political, and climatic factors, as well as industrial structure, living standards, and user behaviors. Indeed, we have shown in this paper that load data contain a very irregular low frequency trend signal, which varies from one region to another, and thus must be affected a number of factors, besides temperature variation. Indeed, one can readily see that the expansion of a city must be also an important factor. Using an adaptive multiscale decomposition, we have further removed the high frequency noise from the detrended data of the power load time series. Detrending combined with denoising clearly extracts the major component of the power load variation, which has very strong spectral peaks corresponding to a period of one day and its harmonics, as well as highly varying amplitude. Further chaos analysis of this major component using scale-dependent Lyapunov exponent (SDLE), which has been shown to be able to unambiguously distinguish chaos from all known types of noise, clearly shows that power load variation is not characterized by a chaotic attractor. To better quantify the random amplitude variations, we then have used adaptive fractal analysis (AFA), which can best extract fractal behaviors from signals with strong oscillatory trends, and shown that load variation has longrange correlations. Amazingly, maxima of the temporal variation of the long-range correlations correspond well with temperature minima, suggesting that the long-range correlations in the load data is stronger in winter than in summer.

Before ending, we make two comments. One is related to the load variation on large time scales. As the winters in the cities of Guilin and Leshan are quite mild, while summers are hot and humid, it is generally thought that the demand on electricity in those two cities will be greater in summer than in winter, as almost every household uses air-conditioning. This is not supported by measured load data. In fact, the load varies quite randomly for the city of Guilin, while it is larger in winter than in summer for the city of Leshan.

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